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ORIGINAL ARTICLE

Optimal design of an activated sludge plant: theoretical analysis

M. A. Islam · M. S. A. Amin · J. Hoinkis

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Abstract The design procedure of an activated sludge plant consisting of an activated sludge reactor and settling tank has been theoretically analyzed assuming that (1) the Monod equation completely describes the growth kinetics of microorganisms causing the degradation of biodegradable pollutants and (2) the settling characteristics are fully described by a power law. For a given reactor height, the design parameter of the reactor (reactor volume) is reduced to the reactor area. Then the sum total area of the reactor and the settling tank is expressed as a function of activated sludge concentration X and the recycled ratio α. A procedure has been developed to calculate X_{opt}, for which the total required area of the plant is minimum for given microbiological system and recycled ratio. Mathematical relations have been derived to calculate the α-range in which X_{opt} meets the requirements of F/M ratio. Results of the analysis have been illustrated for varying X and α. Mathematical formulae have been proposed to recalculate the recycled ratio in the events, when the influent parameters differ from those assumed in the design.

Keywords Activated sludge reactor · Optimal design · Optimal operation · Sludge recycled ratio · Settling tank

List of symbols

a	Constant in settling model (m/day)
A_r	Area of the reactor (m^2)
A_s	Area of the settling tank (m^2)
A_T	Total area (m^2)
F/M	Food to microorganism ratio
F_c	Critical solid flux in settling tank (kg/m^2 day)
F_g	Gravity solid flux in settling tank (kg/m^2 day)
F_L	Limiting solid flux in settling tank (kg/m^2 day)
H_r	Height of the activated sludge reactor (m)
k_d	Endogenous decay coefficient (day^{-1})
n	Constant in settling model
Q_0	Influent wastewater flow rate (m^3/day)
Q_e	Effluent wastewater flow rate (m^3/day)
Q_r	Recycled wastewater flow rate (m^3/day)
Q_w	Withdrawn wastewater flow rate (m^3/day)
r'_g	Net growth rate of the microorganisms (kg/m^3 day)
r_{su}	Substrate utilization rate (kg/m^3 day)
S_0	Influent substrate concentration (kg/m^3)
S	Substrate concentration in the reactor and in the settling tank (kg/m^3)
t	Time (days)
v	Underflow velocity (m/day)
V_r	Activated sludge reactor volume (m^3)
X_0	Activated sludge concentration in the influent (kg/m^3)
X	Activated sludge concentration in the reactor (kg/m^3)
X_e	Activated sludge concentration in the effluent (kg/m^3)
X_r	Activated sludge concentration in the recycled stream (kg/m^3)
X_u	Activated sludge concentration in the underflow stream (kg/m^3)
X_w	Activated sludge concentration in the waste stream (kg/m^3)
Y	Maximum yield coefficient (kg/kg)
α	Sludge recycled ratio
β	Sludge waste ratio
θ	Hydraulic retention time (days)

M. A. Islam (✉) · M. S. A. Amin
Department of Chemical Engineering and Polymer Science, Center for Environmental Process Engineering, Shahjalal University of Science and Technology, Sylhet, Bangladesh
e-mail: islamsust@yahoo.com

J. Hoinkis
Faculty of Electrical Engineering and Information Technology, Karlsruhe University of Applied Sciences, Karlsruhe, Germany

Introduction

Activated sludge plant applies the microbiological process for the degradation of organic pollutants in water. The method does not require chemicals (maybe in insignificant amount). The operational cost is low as compared to the chemical method of treatment. The biodegradation process, however, is slow and the plant requires large footprint of the facilities. In recent years, highly polluting industries have been flourishing in the densely populated countries with poor economy, low-paid working forces and liberal environmental law (in writing or in implementation). Thus, although in those countries, sometimes due to favorable atmospheric conditions, the biological method of wastewater treatment seems to be appropriate, it is hardly chosen as it demands large space. The biological treatment method is sustainable and it is the demand of the day. Therefore, some proper design and operation method has to be developed, which would find the minimum required area for an activated sludge plant and also to ensure the condition for optimal operation of an existing plant, and this requires 'Development of comprehensive mathematical formulations for design and operation'.

Activated sludge system for biological wastewater treatment consists essentially of an activated sludge reactor and a settling tank. Reports on the analysis of the performance of the activated sludge plants based on reactor-settler interaction are rather scarce. Sherrard and Kincannon (1974) proposed a correlation among the mean solid retention time, sludge recycle ratio and sludge concentration factor in secondary settling tank. Riddell et al. (1983) combined the functions of the reactor and the settling tank in order to choose permissible flow rate of an activated sludge plant. Sheintuch (1987) introduced the concept of system response for analyzing the interactions of the functions of the reactor and settling tank. But it was Cho et al. (1996) who described the reactor-settling tank interaction in very comprehensive way. They used the sludge recycle ratio and sludge waste ratio as the operating parameters and obtained the responses of the output variables such as biomass concentration in aerator, dissolved pollutant and solid concentration in the effluent. Diehl and Jeppsson (1998) presented a dynamic simulation model of an activated sludge plant. For describing the continuous sedimentation in the secondary clarifier, a one-dimensional model based on non-linear partial differential equation was proposed. The analysis of the settling process lied on vigorous mathematical treatment of the equation. The Diehl-Jeppsson model, however, does not correlate the dynamics of the reactor to that of the settling tank in a comprehensive way.

In the past decades much research has been conducted on the modeling and simulation of the activated sludge plants (Rigopoulos and Linke 2002; Gernaey et al. 2004; Flores et al. 2005; David et al. 2009), but comprehensive discussion on the interaction between the reactor and the settling tank is still not adequate. Instead, precise models have been proposed for the reactors (Gujer et al. 1999; Hu et al. 2003; Moulleca et al. 2011; Pholchan et al. 2010; Scuras et al. 2001) and also for the settlers (Diehl 2007; Ekama and Marais 2004; Flamant et al. 2004; Vanderhasselt and Vanrolleghem 2000; Zhang et al. 2006; Bürger et al. 2011) independently, and the performances of the units have been discussed. Patziger et al. (2012) studied the settling tank under dynamic load considering both the overall unsteady behavior and the features around the peaks, investigating the effect of various sludge return strategies as well as the inlet geometry on the performance of the tank. Thus a lot of work has been done on the problem, but comprehensive method and mathematical relations considering the reactor-settling tank interactions are yet to be developed for optimal design and operation of a wastewater treatment plant and thus to make ground for the development of computing techniques and software to serve the purpose.

The purpose of the present work is: (1) to develop a method for simultaneous design of a reactor and a settling tank constituting an activated sludge plant for an assigned performance level, (2) to develop an analytical and also graphical method for determining the range of recycle ratio and activated sludge concentration to achieve certain treatment level, (3) to work out a design method, which will ensure minimum footprint for the plant and (4) also to find the conditions of operation of an already functioning plant in the event the input parameters (influent flow rate and substrate concentration) differ from those assumed in designing. The fundamental work of Cho et al. (1996) will be taken as the basis for analysis, in which the authors studied the coupled system of reactor and the settling tank using Monod's simple reaction kinetic model and limit flux theory. In the meantime, Pholchan et al. (2010) reported that the diversity of the microbial communities in the reactors was affected by changes in the operating parameters of the bioreactor. Tench (1994) showed that the biological treatment processes were 'complex systems' where many different kinds of microbes grew and interacted in a dynamic manner. The author concluded that the analysis of the treatment process would be more precise, if all the complexities concerning microorganism activities were taken into account. This paper, however, aims at describing the performance and design of the activated sludge plant in principle, when the operation of the reactor and the settling tank are described by simple equations. Thus, for the simplicity of the present analysis, it is considered that whatever changes may take place with the diversity of the microbial communities in the reactors, the microbiological and settling parameters of the sludge do not change.

In this analysis, the activated sludge plant is considered to be consisted of two units: activated sludge reactor and settling tank. The design parameter of the reactor is volume and that of the settling tank is the area. Assuming a definite height for the activated sludge reactor, the design parameter of both the units has been reduced to area. Thus the optimization criterion is accepted to be the minimum total area (footprint) of the units. It is assumed that the Monod equation fully describes the growth kinetics of microorganism and the settling characteristics of the sludge follows some power law. It is found that in the designing of the interacting reactor-settler system, three variables such the activated sludge concentration X in the reactor, the sludge recycle ratio α and the sludge waste ratio β determine the total area of the plant. For diminishing the number of variables, the value of β was kept constant at 0.01. Contradicting the popular perception that 'an increase in α results in an increase in the biodegradation rate and the overall space required for the plant decreases', it is found that with an increase in α, the total area of the plant monotonously increases. With the sludge waste ratio β kept at constant, the sludge recycle ratio α, however, appears to be the only controllable parameter to maintain the desired level of treatment in the plant when the feed parameters differ from those assumed in the design of the plant. A procedure has been developed for simultaneous design of a reactor and settling tank, ensuring minimum area (footprint) for an activated sludge plant, and also a methodology has been worked out to recalculate the operating sludge recycled ratio in cases when influent parameters differ from those in design.

It should be noted here that the growth kinetic and the settling model used in this work is not the best choice. Analysis would be more precise, if process rate equations are chosen from the Activated Sludge Model series (named as ASM1, ASM2, ASM2d, ASM3), which are the widely accepted models for the design and operation of biological wastewater treatment systems. This is a continuously developing model series, which includes always new elements to entrap new experiences in wastewater treatment (Henze et al. 2002). The ASM1 predicts the performance of single-sludge systems carrying out carbon oxidation, nitrification and denitrification. The ASM2 includes nitrogen and biological phosphorous removal. ASM2d (which is the expanded form of the ASM2) includes the denitrifying activity of the phosphorous accumulating organisms. The ASM3 includes storage of organic substrates as a new process. Iacopozzi et al. (2007) shows that the ASM1, ASM2, ASM2d, and ASM3 are limited to the description of the denitrification on nitrate only, as they present the nitrification dynamics as a single-step process. The authors propose an enhancement to the basic ASM3 model, introducing a two-step model for the process nitrification and thus consider the denitrification on both nitrite and nitrate.

With the inclusion of newer and newer elements in the rate equations of the processes in the reactor, the complexity of the ASM series models increases along with the increase in the preciseness of the predictions. The main purpose of the present work, however, is to illustrate the principle of a methodology for determining the minimum required area for a plant, and hence in order to avoid complexities, the simple Monod equation (for aerobic growth of heterotrophs in excess oxygen as per ASM1 model) has been chosen as the growth kinetics of the activated sludge in the reactor.

Theoretical

The operation of an activated sludge plant is typically presented by a flow diagram as shown in Fig. 1. The wastewater with a substrate concentration of S_0 (kg/m^3) and an activated sludge concentration of X_0 is fed to a reactor at a flow rate of Q_0 (m^3/day). In the reactor, an activated sludge concentration of X (kg/m^3) is maintained in suspension. An aerobic environment is achieved in the reactor by mechanical aeration, which also serves to maintain a completely mixed regime throughout the reaction mass. The substrate undergoes degradation under the action of activated sludge. From the reactor, the water (with the substrate concentration S) along with the activated sludge passes into a settling tank, where the sludge is separated from the treated wastewater by gravitation. An anaerobic environment is maintained in the settling tank, and it is presumed that microbial degradation occurs only in the reactor. Consequently, the substrate concentration at the exit of the reactor is the same as that in the settling tank. The clear portion of the liquid with an activated sludge concentration of X_e is collected from the top of the settling tank with the overflow rate of Q_e (m^3/day). The settled sludge is withdrawn from the bottom of the settling tank. The underflow sludge concentration is X_u. A portion of the settled sludge with concentration X_r is recycled to the reactor at a flow rate of Q_r (m^3/day) to maintain the desired sludge concentration X in the reactor. The excess sludge with the sludge concentration of $X_w = X_r = X_u$ is removed from the system with a flow rate of Q_w (m^3/day). The substrate concentration S in the clarified liquid must conform to the standard imposed by the Department of Environment.

Design equation for the reactor

The following assumptions have been made in the design:

(1) The growth kinetics follows the simple Monod equation,
(2) The flow behavior in the reactor is assumed to be completely mixed,

(3) The reactor is operating under steady state, and
(4) The activated sludge concentration in the influent (kg/m^3), $X_O = 0$.

Referring to Fig. 1, the overall mass balance equation for the plant with respect to the microorganisms and the substrate can be written as follows:

$$V_r(dX/dt) = Q_0X_0 - Q_wX_w - Q_eX_e + V_r r'_g \tag{1a}$$

$$V_r(dS/dt) = Q_0S_0 - Q_wS - Q_eS + V_r r_{su} \tag{1b}$$

where V_r (m^3) is the reactor volume, r'_g is the net growth rate of the microorganisms (kg/m^3 day), r_{su} is the substrate utilization rate (kg/m^3 day) and t is the time (d).

Following Monod kinetics of growth rate for the microorganisms r'_g can be expressed as follows (Metcalf and Eddy 1998):

$$r'_g = -Yr_{su} - k_dX \tag{2}$$

where Y (kg/kg) is the maximum yield coefficient (defined as the ratio of the mass of the new cells formed to the mass of the substrate consumed, measured during any finite period of logarithmic growth), k_d is the endogenous decay coefficient (d^{-1}).

If all the food in the system is converted to biomass, relationship between food utilization rate (dS/dt) and biomass utilization rate (dX/dt) can be written as follows:

$$-\frac{dS}{dt} = \frac{1}{Y}\frac{dX}{dt}. \tag{3}$$

Combining Eqs. (1–3) for steady state condition, we obtain

$$\theta = \frac{1}{k_d}\left[\frac{Y(S_0 - S)}{X} - \frac{\beta(1+\alpha)}{\alpha+\beta}\right] \text{ with } \alpha = Q_r/Q_0, \text{ and } \beta = Q_w/Q_0 \text{ and } \theta = V_r/Q_0 \tag{4}$$

where θ is the hydraulic retention time, α is the sludge recycle ratio and β is the sludge waste ratio.

Equation (4) gives the required volume of the activated sludge reactor for given value of the operating parameters α, β, S_0 and S, and the assumed sludge concentration X in the reactor. This equation clearly indicates that with an increase in the activated sludge concentration in the reactor, the volume of the reactor required to achieve assigned treatment level decreases. From the mathematical viewpoint, the sludge concentration X may assume any value higher than zero, resulting in positive, negative or zero reactor volume. But for ensuring proper environment for biochemical reaction in the reactor, some restrictions must be imposed to X in order to maintain the 'food to microorganism ratio' F/M in the reactor in a defined range. The restriction is given by the following relation:

$$f_{min} \le F/M = \frac{S_0}{\theta X} \le f_{max} \tag{5}$$

where f_i is some assigned value to F/M ratio. The F/M ratio is usually recommended to be in the range of (0.2, 1.0) (Metcalf and Eddy 1998). Combining Eq. (4) and Equation/Inequality (5), we obtain

$$\frac{\alpha+\beta}{\beta(1+\alpha)}\left[Y(S_0 - S) - \frac{S_0k_d}{f_{min}}\right] \le X \le \frac{\alpha+\beta}{\beta(1+\alpha)}\left[Y(S_0 - S) - \frac{S_0k_d}{f_{max}}\right] \tag{6}$$

with

$$X_{min} = \frac{\alpha+\beta}{\beta(1+\alpha)}\left[Y(S_0 - S) - \frac{S_0k_d}{f_{min}}\right] \text{ and } X_{max} = \frac{\alpha+\beta}{\beta(1+\alpha)}\left[Y(S_0 - S) - \frac{S_0k_d}{f_{max}}\right], \tag{6a}$$

Thus, Equation/Inequality (6) gives the permissible upper and lower limit of the activated sludge concentration (X_{max}

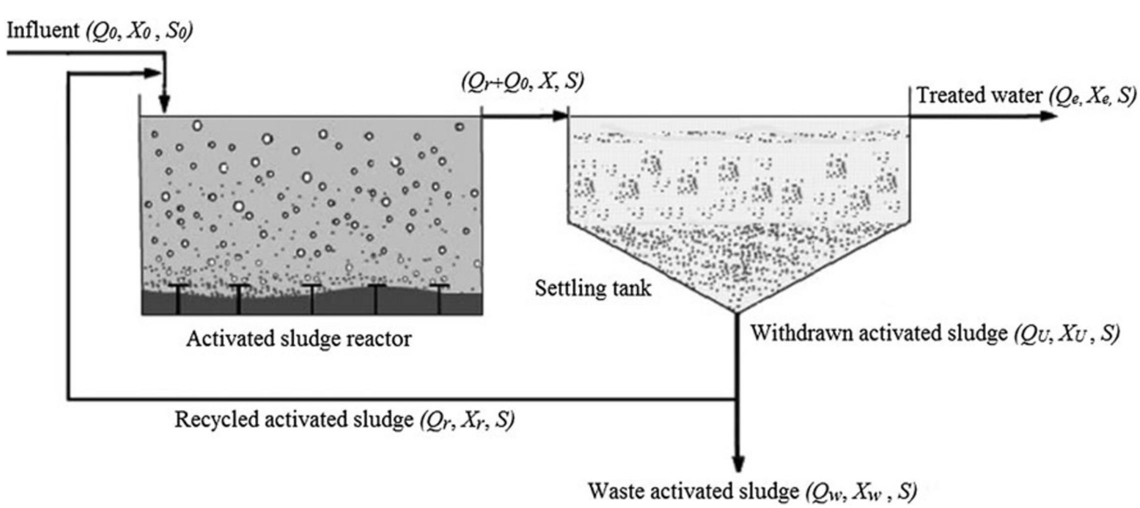

Fig. 1 Technological sketch of a typical activated sludge plant

and X_{min}) for given sludge recycled ratio α and sludge waste ratio β, ensuring the defined range of F/M ratio.

Design equation for the settling tank

Conventional settler models have been based on the solids flux theory of (Coe and Clevenger 1916) which stated that the total solids flux equals the sum of the solid flux due to the gravity settling and the bulk downward movement of the liquid. The graphical method for the determining the design parameter (area) of the settler is presented in Fig. 2. A gravity settling flux F_g versus sludge concentration X curve is drawn. For the desired underflow concentration X_u, a tangent is drawn on the curve from the point $A(X_u, 0)$, which touches the curve at the point $B(X_c, F_c)$ and intersects the solid flux axis at the point $C(0, F_L)$. X_c is the critical sludge concentration determining the limiting flux F_L (total flux resulted from the gravity settling rate and applied sludge-withdrawal rate) for the underflow concentration X_u (Metcalf and Eddy 1998). Then the required settling area (A_s) for $X_e = 0$ is calculated from the following relation:

$$A_s = (Q_0 + Q_r)X/F_L = (1+\alpha)Q_0X/F_L \quad (7)$$

Analytical expression for the limiting flux, F_L:

A number of empirical equations are available in literature describing the relation between the settling velocity and the concentration of the sludge (Smollen and Ekama 1984; Vesilind 1968; Islam and Karamisheva 1998). For the simplicity of the present analysis, the following assumptions have been made:

(1) The settling rate of the floc follows power law of the type $v_s = aX^{-n}$
(2) The flow behavior in the settling tank is assumed to be ideally plug flow type; vertical mixing as well any concentration-variation in the radial direction is ignored,
(3) The settling tank is operating under steady state,
(4) The growth rate in the settling tank is zero, and
(5) The activated sludge concentration in the effluent (kg/m^3), $X_e = 0$.

The gravity settling flux F_g versus sludge concentration X curve in Fig. 2 is also described by some power law as described by Eq. (8).

$$F_g = Xv_s = X.aX^{-n} = aX^{-n+1} \quad (8)$$

where F_g is the gravity flux (kg/m^2 day), and a (m/day) and n are empirical parameters characterizing the settling properties of the sludge. The point B belongs to both the tangent and the gravity settling curve. Hence, following Eq. (8), its coordinates are (X_c, F_c) with $F_c = aX_c^{-n+1}$ and the slope of the tangent at the point is given by Eq. (9).

$$\text{The slope of the tangent } AC \text{ at the point } B = \left(\mathrm{d}F_g/\mathrm{d}X\right)X = X_c = -a(n-1)X_c^{-n} \quad (9)$$

Expressing the slope of the straight line AC in terms of the coordinates of the points A, B and C, and then equating it with that in Eq. (9), we have

$$\text{The slope of the straight line } AC = \frac{-F_L}{X_u} = \frac{F_c}{X_c - X_u} = -a(n-1)X_c^{-n} \text{ with } F_c = aX_c^{-n+1} \quad (10)$$

Solving the relations in Eq. (10) with respect to X_u, we obtain

$$X_c = \frac{n-1}{n}X_u \quad \text{and} \quad F_L = a(n-1)\left(\frac{n-1}{n}\right)^{-n}.X_u^{-n+1} \quad (11)$$

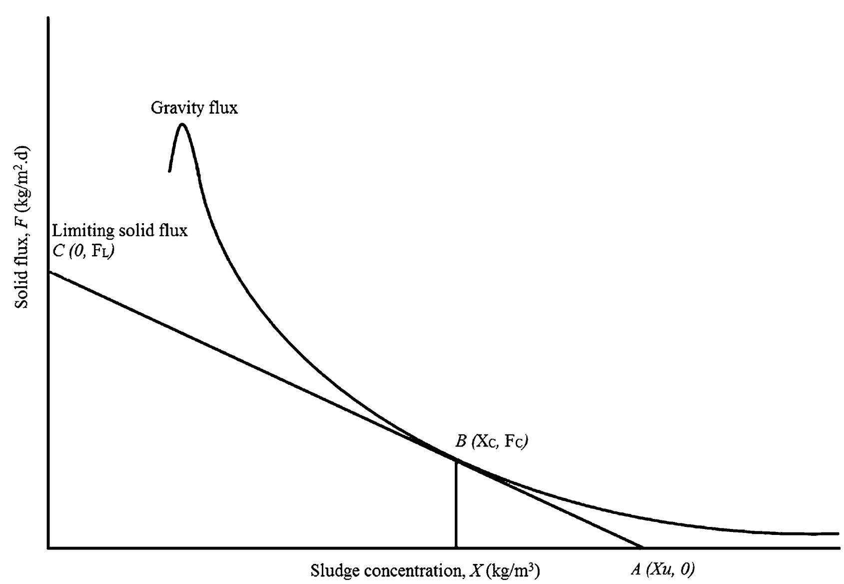

Fig. 2 Solid flux versus sludge concentration plot. The description is given in the text

The mathematical expression for the total limiting flux F_L from Eq. (11) can be substituted in Eq. (7) to give the equation for determining the settling area A_s in terms of X and X_u.

From the material balance around the settling tank in Fig. 1 with respect to the activated sludge concentration, we obtain

$$(Q_0 + Q_r)X = Q_u X_u \quad \text{with} \quad Q_u = Q_r + Q_w \quad \text{and} \quad X_e = 0. \tag{12}$$

Expressing in terms of the sludge recycled and waste sludge ratio, Eq. (12) is reduced to Eq. (13) giving the relation between X and X_u.

$$X_u = \frac{1+\alpha}{\alpha+\beta}X. \tag{13}$$

Finally, Eqs. (7, 11, 13) are combined together to give the working equation (Eq. 14) for determining the settling area A_s.

$$\frac{A_s}{Q_0} = \frac{(1+\alpha)^n}{\gamma(\alpha+\beta)^{n-1}}X^n \quad \text{with} \quad \gamma = a(n-1)\left(\frac{n}{n-1}\right)^n. \tag{14}$$

Optimal design

The design parameter for the reactor is the volume (V_r) and that for the settling tank is the area (A_s). Considering the minimum footprint of the facilities as the criterion for optimization, the design parameter of the reactor has to be reduced to 'area'. This is achieved by assigning a definite value H_r to the height/depth of the reactor to be designed. Then the design equation (Eq. 4) of the reactor in term of the volume V_r can be rewritten in term of the reactor area A_r as follows:

$$\frac{A_r}{Q_0} = \frac{1}{H_r k_d}\left[\frac{Y(S_0 - S)}{X} - \frac{\beta(1+\alpha)}{\alpha+\beta}\right]. \tag{15}$$

Adding Eq. (14) to Eq. (15), we get the expression for the total design area A_T of the facilities. Thus,

$$\frac{A_r}{Q_0} + \frac{A_s}{Q_0} = \frac{A_T}{Q_0} = \frac{1}{H_r k_d}\left[\frac{Y(S_0 - S)}{X} - \frac{\beta(1+\alpha)}{\alpha+\beta}\right] + \frac{(1+\alpha)^n}{\gamma(\alpha+\beta)^{n-1}}X^n. \tag{16}$$

Now the task is to find those values of the parameters X, α and β for which the value of A_T will be minimum subjected to the restriction imposed by Equation/Inequality (6). The analysis in the present work is done for a constant value of β equal to 0.01. Thus, A_T ceases to depend on the waste sludge ratio β. Therefore, only the following equations should be solved simultaneously (if there is any minimum at all).

$$\frac{\partial A_T}{\partial X} = \frac{\partial A_T}{\partial \alpha} = 0 \tag{17}$$

Optimum value of the activated sludge concentration, X_{opt}

Differentiating Eq. (16) with respect to X, we have

$$\frac{1}{Q_0}\frac{\partial A_T}{\partial X} = \frac{1}{Q_0}\left(\frac{\partial A_r}{\partial X} + \frac{\partial A_s}{\partial X}\right) \tag{18}$$

with $\frac{1}{Q_0}\frac{\partial A_r}{\partial X} = -\frac{1}{H_r k_d}\frac{Y(S_0 - S)}{X^2}$ and $\frac{1}{Q_0}\frac{\partial A_s}{\partial X} = \frac{(1+\alpha)^n}{\gamma(\alpha+\beta)^{n-1}}nX^{n-1}$.

Equation (18) shows that $\partial A_r/\partial X$ is negative, but $\partial A_s/\partial X$ is positive for any value of X. Thus, for some given value of α and β, the reactor area (correspondingly reactor volume) decreases as the activated sludge concentration increases. The settling area, however, increases with the increase in the activated sludge concentration. Naturally, it is expected that the total area A_T will pass through minimum for some optimal value of $X = X_{opt}$.

Equating $\partial A_T/\partial X$ to zero and doing some algebraic manipulation, we obtain:

$$X_{opt} = \left[\frac{\gamma Y(S_0 - S)}{H_r k_d n}\cdot\frac{(\alpha+\beta)^{n-1}}{(1+\alpha)^n}\right]^{1/(n+1)} \tag{19}$$

Equation (19) gives an optimal value of X for assigned value to $\alpha \geq 0$ and $\beta \in (0, 1)$. All values of X_{opt}, however, cannot be used effectively. The effective values are those, which satisfy Equation/Inequality (6).

Optimum value of the sludge recycle ratio, α_{opt}

Differentiating Eq. (16) with respect to α, we have

$$\frac{1}{Q_0}\frac{\partial A_T}{\partial \alpha} = \frac{1}{Q_0}\left(\frac{\partial A_r}{\partial \alpha} + \frac{\partial A_s}{\partial \alpha}\right) \tag{20}$$

with

$$\frac{1}{Q_0}\frac{\partial A_r}{\partial \alpha} = \frac{1}{H_r k_d}\cdot\frac{\beta(1-\beta)}{(\alpha+\beta)^2} \tag{20a}$$

and

$$\frac{1}{Q_0}\frac{\partial A_s}{\partial \alpha} = \frac{(1+\alpha)^{n-1}}{\gamma(\alpha+\beta)^n}\xi X^n \quad \text{with} \quad \xi = \alpha + 1 - n(1-\beta) \tag{20b}$$

It appears that $\partial A_r/\partial \alpha$ is always positive and the reactor area (correspondingly volume) increases as the sludge recycle ratio α increases. $\partial A_s/\partial\alpha$, on the other hand, may assume positive, negative or zero-value depending on the sign of the parameter ξ. In the range of low values of α, $\partial A_r/\partial\alpha$ may become negative and the settling area decreases as the sludge recycled ratio increases. For high values of α, the value of ξ, becomes positive and the settling area increases as the sludge recycled ratio increases.

Condition of absolute minimum for A_T

The question is whether there exists any value of X and α, for which both $\partial A_T/\partial X = 0$ and $\partial A_T/\partial\alpha = 0$. For the purpose, Eq. (19) is substituted in Eq. (20) and equating $\partial A_T/\partial\alpha$ to zero, we have:

$$\frac{1}{H_r k_d}\cdot\frac{\beta(1-\beta)}{(\alpha+\beta)^2} + \xi\left[\frac{Y(S_0-S)}{H_r k_d}\cdot\frac{1}{n\gamma^{1/n}(\alpha+\beta)^2(1+\alpha)^{1/n}}\right]^{n/(n+1)} = 0 \tag{21}$$

The first term on the left hand side of Eq. (21) is always positive. The second term on the left hand side may be positive, negative or zero depending on the sign of the parameter ξ. Thus, if Eq. (21) has got some solution for $\alpha = \alpha_{opt}$, then putting that value of α in Eq. (19), the value of X_{opt} can also be calculated, and finally, the minimum total area $(A_T/Q_0)_{min}$ can be calculated from Eq. (16). Attempts have been made to solve Eq. (21) by trial and error method, but it was found that it had no solution and for any value of α under study, $\partial A_T/\partial\alpha$ is greater than zero (illustrated later in Fig. 5), which will mean that the total area increases monotonously as α increases.

Illustration of the model

The data used for the illustration of the model are summarized in Table 1. The microbiological as well as settling parameters chosen in Table 1 are similar to those reported in literature (Cho et al. 1996; Smollen and Ekama 1984). For sample calculations (Smollen and Ekama 1984) used 325 m/day as the value of a. For the value of k_d Y, a and n, Cho et al. 1996 used 0.06 day^{-1}, 0.6 kg/kg, 375 m/day and 2.3, respectively.

The sludge concentration range for maintaining F/M ratio within a defined range is presented as a function of the sludge recycled ratio α in Fig. 3. The F/M ratio is maintained in the zone entrapped by the upper and lower limit of permissible sludge concentration (calculated by Eq. 6) The optimum activated sludge concentration, X_{opt} (calculated by Eq. 19) is also presented on the same plot. Obviously, the usable X_{opt}-values are those which lie in the permissible zone.

The upper and lower limit of the permissible sludge concentration zone, respectively, X_{max} and X_{min}, monotonously increases with the increase in α. Theoretically, the X_{opt}—curve passes through a maximum (for the given data set, the maximum X_{opt} is 3.392 at $\alpha = 1.5$). But in Fig. 3 the extremum is merely distinguishable from neighboring points to be encountered for practical purposes, as X_{opt} initially increases with the increase in α and then for a wide range of α, the variation in X_{opt} is negligible. For the given data set, the X_{opt} may be assumed to be practically constant at 3.38 ± 0.01 in the α-range of 1.1–2.0. The point of intersection between the X_{opt} and X_{max}-curves, and X_{opt} and X_{min}-curves (solving Eqs. 6, 19), respectively, gives the lower and upper limit of α (Eqs. 22, 23) to be used for the optimal design of the reactor maintaining the desired range of F/M ratio.

$$\alpha_{min} = \left[-(2\beta - m_1) + \sqrt{(2\beta - m_1)^2 - 4(\beta^2 - m_1)}\right]/2 \tag{22}$$

$$\alpha_{max} = \left[-(2\beta - m_2) + \sqrt{(2\beta - m_1)^2 - 4(\beta^2 - m_2)}\right]/2 \tag{23}$$

$$\text{With,}\quad m_1 = \frac{\beta^{n+1}\gamma Y(S_0-S)/(H_r k_d n)}{[Y(S_0-S) - S_0 k_d/f_{max}]^{n+1}} \quad \text{and} \quad m_2 = \frac{\beta^{n+1}\gamma Y(S_0-S)/(H_r k_d n)}{[Y(S_0-S) - S_0 k_d/f_{min}]^{n+1}} \tag{23a}$$

The reactor area per unit flow rate, A_r/Q_0 (equivalent to V_r/H_rQ_0), the settling area per unit flow rate, A_s/Q_0 and the

Table 1 Data for the illustration of the model and plant designtgroup cols="2">

Water parameter	
Influent flow rate (Q_0)	20,000 m^3/day
Inlet substrate concentration (S_0)	0.25 kg/m^3
Outlet substrate concentration (S)	6.0×10^{-3} kg/m^3
Microbial parameters	
Endogenous decay coefficient (k_d)	0.06 d^{-1}
Maximum yield coefficient (Y)	0.5 kg/kg
Sludge settling characteristics	
Empirical coefficients	
a	350 m/day
n	2.5
Operational parameters	
Sludge recycled ratio (α)	Variable
Sludge waste ratio (β)	0.01
Activated sludge concentration (X)	Variable
Additional data	
Assigned value to reactor height/depth (H_r)	4 m

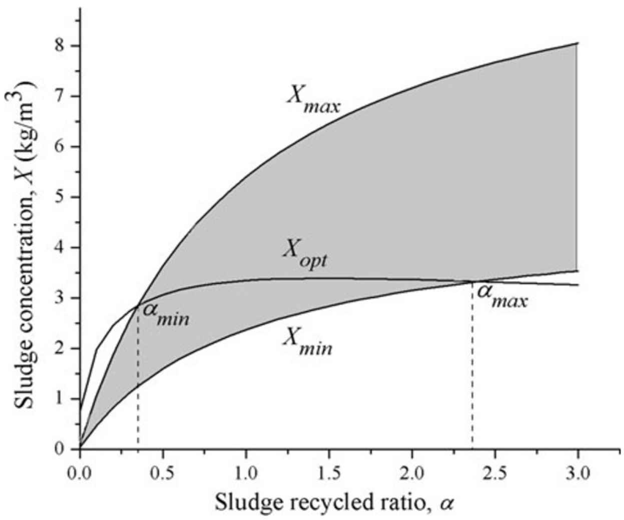

Fig. 3 The optimum activated sludge concentration, X_{opt} (calculated by Eq. 19) as a function of the sludge recycled ratio α. X_{max} and X_{min} (calculated by Eq. 6) are respectively the upper and lower boundary of the permissible sludge concentration zone (*shaded area*). The parameters for calculating the quantities are described in Table 1

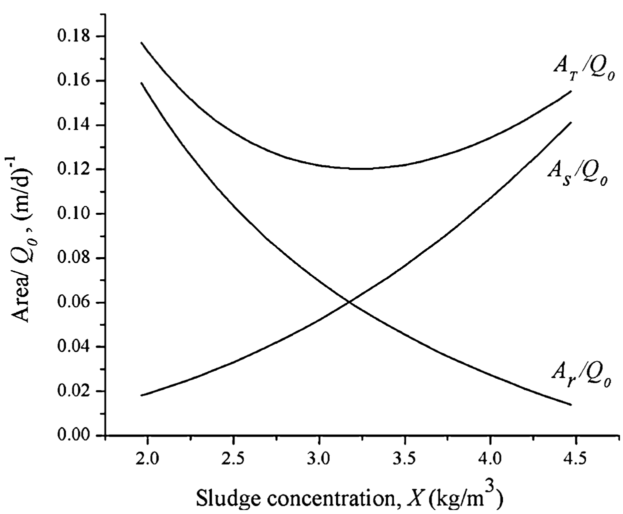

Fig. 4 Area per unit flow rate versus activated sludge concentration X for $\alpha = 0.7$. (1) Reactor area per unit flow rate, A_r/Q_0 (or V_r/H_rQ_0 with $H_r = 4$ m), (2) the settling area per unit flow rate, A_s/Q_0 and (3) total area per unit flow rate A_T/Q_0

total area per unit flow rate A_T/Q_0 are presented in Fig. 4 as a function of X for $\alpha = 0.7$. The calculation of the quantities is done by Eqs. (14–16). The data for the estimation are described in Table 1.

As seen from Fig. 4 the reactor area (consequently volume) decreases as the activated sludge concentration X increases. The settling area, however, increases as the activated sludge concentration increases. Reasonably, the total area for the facilities passes through minimum. For the given data set in Table 1 and for $\alpha = 0.7$, the minimum total area is obtained at $X = 3.23$ kg/m^3, which is the same as the optimum sludge concentration X_{opt} predicted by Eq. (19) (and also as that illustrated in Fig. 3).

The reactor area per unit flow rate, A_r/Q_0 (equivalent to V_r/H_rQ_0), the settling area per unit flow rate, A_s/Q_0 and the total area per unit flow rate A_T/Q_0 are presented in Fig. 5 as a function of α. The calculation of the quantities is done by Eqs. (14–16) for $X = 3.36$ kg/m^3. The required data for the estimation are described in Table 1.

As seen from Fig. 5, the reactor area (consequently volume) increases as the sludge recycled ratio α increases. The increment rate, however, gradually decreases. Theoretically, the settling area versus sludge recycled ratio plot passes through minimum at $\xi = 0$, which corresponds to $\alpha = 1.46$ (see Eq. 20b). The minimum value (which is merely distinguishable from other points to be used for practical purposes) of the settling area at $\alpha = 1.46$ is calculated to be $A_s/Q_0 = 0.060$ (m/day)$^{-1}$. Practically, for $\alpha < 1.0$ the settling area decreases as α increases, and in the range of $\alpha = 1.1$–2.0, the settling area becomes practically independent of α. In this range of α, A_s/Q_0 may be assumed to be constant at 0.061 ± 0.001 (m/day)$^{-1}$ and the minimum value lies within this range. The total area, however, monotonously increases with the increase of α without showing any trend of decreasing. This means that the increase in α will always lead to the increase in the total area of the facilities. The effect of X on the total area of the facilities of an activated sludge plant for different sludge recycled ratio α is summarized in Fig. 6. The total area A_T is calculated by Eq. (16), but only that section of the curves is drawn, which meets the defined *F/M* ratio. As predicted by Eq. (19) and also illustrated in Fig. 3, the optimal sludge concentration for $\alpha \in (0.7, 2.0)$ is (3.35 ± 0.01) kg/m^3. For a given X, the total area increases as the sludge recycled ratio α increases and this sequence is valid for the minimum of the total area at different α.

Design procedure

From the present analysis, it becomes evident that for the system with known microbiological and settling parameters, the following sequence will be maintained for the design of the reactor and settling tank:

(1) Choose a height, H_r, for the reactor
(2) Choose waste sludge ratio, β
(3) Define the range of *F/M* ratio
(4) Determine the value of α_{min} and α_{max} analytically (Eqs. 22, 23) or graphically from the plots analogous to those in Fig. 3.
(5) Choose a suitable sludge recycled ratio $\alpha_{min} < \alpha < \alpha_{max}$
(6) Determine the permissible X-range for which the *F/M* ratio is maintained (Use Equation/inequality (6) for the purpose)

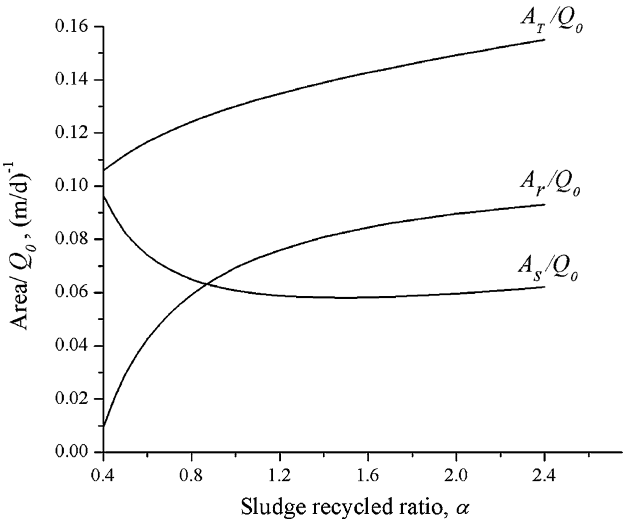

Fig. 5 Area per unit flow rate versus recycled sludge ratio α for $X = 3.35$. (1) Reactor area per unit flow rate, A_r/Q_0 (or V_r/H_rQ_0 with $H_r = 4$ m), (2) the settling area per unit flow rate, A_s/Q_0 and (3) total area per unit flow rate A_T/Q_0

(7) Calculate the optimum value of X by Eq. (19)
(8) Calculate the area of the settling tank by Eq. (14)
(9) Calculate the reactor area by Eq. (15) or volume by Eq. (4).

The total area calculated by the above procedure will give the minimum area for the facilities for the chosen α, β and H_r. There remains, however, a very confusing element in the design as to why the designer should choose higher α with complete awareness that it leads to higher area, while the lower α with lower total area could ensure the same performance. To find the answer of the question, the role of α should be analyzed in maintaining the stable performance of the plant in the event the feed parameters deviate from those assumed in the design. In the next sections, how and to what extent, a plant built with the proposed design method could be adjusted to the influent parameters will be discussed.

Adjustability of the designed activated sludge plant

In the previous sections, analysis has been done on the design of an activated sludge plant consisting of an activated sludge reactor and a settling tank. Now let's assume that for the specified microbiological and settling characteristics in Table 1, three activated sludge plants P-I, P-II and P-III have been designed, the design parameters of which are described in Table 2. In designing of the three plants the sludge recycled ratio α has been chosen in such a way that for the plant P-I, it is almost equal to α_{min}, for the plant P-III, it is much higher and for the plant P-II—some value in between.

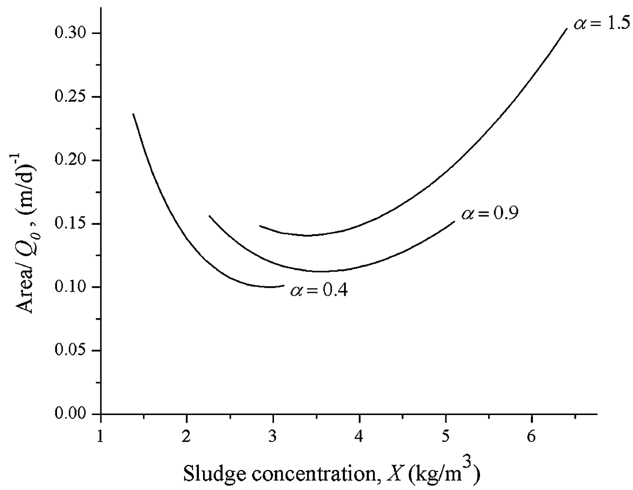

Fig. 6 Total area (footprint of the facilities) per unit flow rate A_T/Q_0 versus activated sludge concentration X for the different sludge recycled ratio

Let the three plants be working in complete mix regime. If the influent parameters (flow rate and influent substrate concentration) do not vary from those assumed in designing and also the microbiological and settling parameters of the sludge do not change, then the operator simply has to maintain the designed sludge recycled ratio $\alpha = 0.35$, 0.50 and 0.90, respectively, for the three plants and the desired performance ($S = 6.0 \times 10^{-3}$ kg/m^3) will be achieved. The steady state activated sludge concentration X in all the three reactors will be maintained as that assumed in design spontaneously (without direct intervention from the operator). The question is how the performance of the plant be maintained at the desired level if the flow rate and/or the influent substrate concentration differ within certain range from that assumed in designing. The operator has got only one operating parameter to control and that is the sludge recycled ratio α. How can the new operating sludge recycled ratio α be chosen (maintaining the F/M ratio) such that the treatment level of the plant remains the same as that assumed in designing?

Recalculation of α for new influent flow rate and substrate concentration

Let the new influent flow rate is $Q_{ni} = \lambda_q Q_0$ and the new influent substrate concentration is $S_{ni} = \lambda_s S_0$; where λ_q and λ_q are respectively the variation factor from the corresponding design parameter. Making the material balance for the fluid stream under steady state, it can be shown that

$$Q_{nr} = \lambda_q Q_r,\ Q_{nw} = \lambda_q Q_w,\ \ \alpha = Q_{nr}/Q_{ni} = Q_r/Q_0 \quad \text{and} \quad \beta = Q_{nw}/Q_{ni} = Q_w/Q_0 \tag{24}$$

where Q_{nr} and Q_{nw} are respectively the new recycled flow rate and the new waste flow rate. The sludge recycled ratio

α and the waste sludge ratio β preserves the same meaning as defined in Eq. (4) To recalculate the operating parameter α_{re}, Eqs. (5, 14, 15) should be rewritten as Eqs. (25–27) under the changed conditions and if some solution is available, the performance of the plant will be at the desired level. Thus, the new operating parameter α_{re} must satisfy the following relations:

$$f_{\min} \le F/M = \frac{S_0}{\theta X} = \frac{Q_{ni} S_{ni}}{V_r X} = \frac{\lambda_q Q_0 \lambda_s S_0}{V_r X} \le f_{\max} \tag{25}$$

$$\frac{A_s}{Q_{ni}} = \frac{A_s}{\lambda_q Q_0} = \frac{(1+\alpha_{re})^n}{\gamma(\alpha_{re}+\beta)^{n-1}} X_{re}^n \text{ with } \gamma = a(n-1)\left(\frac{n}{n-1}\right)^n \tag{26}$$

and

$$\frac{A_r}{Q_{ni}} = \frac{A_r}{\lambda_q Q_0} = \frac{1}{H_r k_d}\left[\frac{Y(S_{ni}-S)}{X_{re}} - \frac{\beta(1+\alpha_{re})}{\alpha_{re}+\beta}\right] \text{ with } S_{ni} = \lambda_s S_0 \tag{27}$$

where X_{re} is the reestablished steady state activated sludge concentration under the new operating conditions.

It should be noted here that for the maintenance of desired *F/M* level in the reactor during design, the *X*-range was bounded by complicated inequalities defined by the restriction Equation/inequality (6). During operation, the restriction Equation/inequality (23) has got much simpler form. This is so, as during the design, V_r is unknown and varies with α, but in operation V_r is known and independent of α.

Table 2 Design parameters for three activated sludge plants I, II and III

Water parameters			
Influent flow rate (Q_0)	20,000 m^3/day		
Inlet substrate concentration (S_0)	0.25 kg/m^3		
Outlet substrate concentration (S)	6.0×10^{-3} kg/m^3		
Assumed parameters	Plant P-I	Plant P-II	Plant P-III
Sludge recycled ratio (α)	0.35	0.50	0.90
Sludge waste ratio (β)	0.01	0.01	0.01
Activated sludge concentration (X), kg/m^3	2.85	3.07	3.32
Height of the reactor (H_r), m	4	4	4
Calculated parameters			
Volume of the Reactor (V_r), m^3	1,776	3,444	5,364
Area of the reactor (A_r), m^2	444	861	1,341
Area of the settling tank (A_s), m^2	1,428	1,328	1,205
Total area of the Plant (A_T), m^2	1,872	2,189	2,546

Solution of the system of Eqs. (25–27)

The *F/M* -restriction Equation/inequality (25) is rearranged as follows:

$$\frac{\lambda_q \lambda_s Q_0 S_0}{V_r f_{\max}} \le X \le \frac{\lambda_q \lambda_s Q_0 S_0}{V_r f_{\min}}. \tag{28}$$

Rearranging Eq. (27), X_{re} may be expressed as follows:

$$X_{re} = Y(\lambda_s S_0 - S) \cdot \left[\frac{H_r k_d A_r}{\lambda_q Q_0} + \frac{\beta(1+\alpha_{re})}{\alpha_{re}+\beta}\right]^{-1}. \tag{29}$$

Substituting Eq. (29) into Eq. (26), we have

$$\frac{A_s}{\lambda_q Q_0} = \frac{(1+\alpha_{re})^n}{\gamma(\alpha_{re}+\beta)^{n-1}} \cdot \left[\frac{Y(\lambda_s S_0 - S)}{\frac{H_r k_d A_r}{\lambda_q Q_0} + \frac{\beta(1+\alpha_{re})}{\alpha_{re}+\beta}}\right]^n. \tag{30}$$

Recalculation procedure for α_{re}:

Step-1 calculate the *X*-range in compliance with $f_{\min} \le F/M \le f_{\min}$ using Equation/Inequality (28);

Step-2 solve Eq. (30) for $\alpha_{re} > 0$ by trial and error method;

Step-3 calculate X_{re} using Eq. (29) and check whether it satisfies the *X*-range calculated in the Step 1. If yes, then accept the α_{re} as the operating parameter.

In the events, Monod equation describes the biodegradation process and the settling rate follows the power law, whatever might be the values of microbiological and settling parameter, Eqs. (25–27) can be solved for determining the operation parameter α. Even if no concrete method is strictly followed in the design of the reactor and the settling tank, Eqs. (25–27) would again give the correct α (if found such satisfying Eqs. 25–27) for operation.

Illustration of the new operating conditions

Let's determine the operating sludge recycled ratio for the three plants described in Table 2 for two cases: (1) The influent flow rate is different from that used in design Q_0 ($\lambda_q \neq 1$), but the influent substrate concentration remains the same as S_0 ($\lambda_s = 1$) and (2) The influent flow rate is the same as that used in design Q_0 ($\lambda_q = 1$), but the influent substrate concentration is different from that used in design S_0 ($\lambda_s \neq 1$). For $\lambda_q = \lambda_s = 1$, the plants carries the organic load as designed (which is equal to Q_0 S_0). For λ_q and/or $\lambda_s > 1$, the plants carry the organic load higher than that designed for, and for λ_q and/or $\lambda_s < 1$, the plants carry lower organic load than that designed for.

Case 1: ($\lambda_q \neq 1$, $\lambda_s = 1$)

In order to achieve the same performance as designed, the sludge recycled ratio α_{re} and the activated sludge concentration X_{re} have been recalculated by Eqs. (29) and (30), respectively. Then α_{re} is presented in Fig. 7 as the function of the flow rate variation factor λ_q; but only that section of the curves is drawn, for which the value of X_{re} meets the requirement to *F/M* ratio as given by the restriction Eq. (28).

Figure 7 shows that choosing appropriate operating sludge recycle ratio α, for $\lambda_q > 1$ the recycled ratio of the plants P-II and P-III can be readjusted to the new influent flow rate and ensure the same performance as designed. This is, however, not possible for the plant P-I, which has been designed with $\alpha = 0.35$ (almost equal to $\alpha_{min} = 0.349$). It appears that the plant P-I can not carry influent load higher than Q_0 (with substrate concentration S_0). The plants P-II and P-III have got some capacity in reserve. The plant P-III being designed with $\alpha = 0.9$ has got higher reserve (up to $\lambda_q = 2.0$) than the plant P-II designed with $\alpha = 0.5$ (up to $\lambda_q = 1.8$).

For $\lambda_q < 1$, the sludge recycle ratio of all the three plants can again be readjusted. But the recalculated α for the plants P-II and P-III appears to be unexpectedly high to be applied for influent flow less than that foreseen in design and some alternative/additional solution is to be sought for such cases.

Case 2: $\lambda_q = 1$, $\lambda_s \neq 1$

Equations (25–27) have been solved for different values of λ_s for the three plants, and the recalculated operating

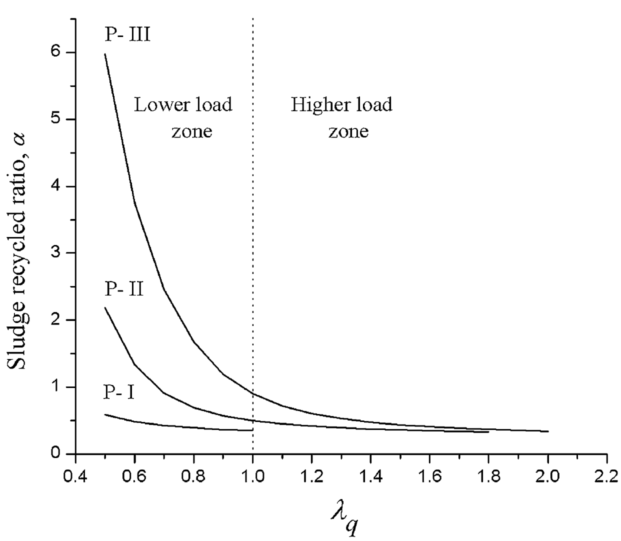

Fig. 7 Recalculated operating parameterα_{re} versus flow rate variation factor λ_q for the three plants P-I, P-II and P-III described in Table 2

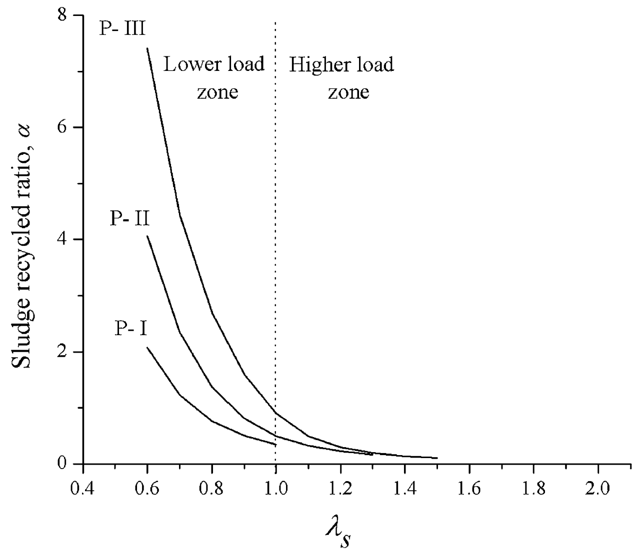

Fig. 8 Recalculated operating parameterα_{re} versus substrate concentration-variation factor λ_s for the three plants P-I, P-II and P-III described in Table 2

parameter α_{re} has been presented in Fig. 8 as the function of the substrate concentration-variation factor λ_s.

Figure 8 shows that the effect of λ_s is similar to that of λ_q. Although not completely proportional both the variation factors have unidirectional effect on the recalculated operating parameters. Both Figs. 7 and 8 clearly indicate that the 'optimal design' is not enough for optimal operation of a plant in cases, when the load deviates from that assumed in the design.

For λ_q, $\lambda_s < 1$, the recalculated recycled ratio is not appealing to be applied for practical purposes. It should be remembered that the design equation was derived in terms of area per unit flow rate. Therefore if the design area/volume is partitioned and is used partially in necessity, the area per unit flow rate can be kept nearly equal to that assumed in design and the recalculated α will be similar to that in the design. Thus, the distribution of the reactor and the settling tank into several parallel units might be helpful to handle to organic load rate lower than that in the design. More study is required on series and parallel arrangement of the units to make a conclusion in this respect.

Conclusions

A design procedure has been developed, which would ensure the minimum area (footprint) for an activated sludge plant consisting of a reactor and settling tank. It is found that the reactor volume increases as the sludge recycled ratio α increases. The total area of the plant also increases with an increase in α. A methodology has also been worked out to recalculate the operating sludge recycled ratio in cases when influent parameters differ from those assumed in the design.

In fact, this work describes the principle of the development of a methodology and it is illustrated with the assumption that the growth kinetics and the settling characteristics of the activated sludge are described by simple Monod equation and a power law, respectively. More precise result is expected, if the methodology is applied to more precise rate equations as recommended by the ASM models for a given case. Also, the assumption of ideal flow behavior in the reactor as well as in the settling tank will bring some error in the estimation. A correction factor could be introduced to account for the deviation the flow pattern from ideality. The procedure and mathematical formulations developed in this work for design and operation can be used in the development of software for the purpose.

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